Quantum models based on finite groups: statistical description

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The trajectory of a quantum system is a sequence of unitary evolutions of vectors in a Hilbert space interspersed with observations (measurements). The observations are described mathematically as orthogonal projections of the vectors on subspaces that are specified by measuring devices. The result of quantum observation is random and its statistics is described by a probability measure defined on subspaces of the Hilbert space. Gleason's theorem gives a general construction of all possible probability measures on subspaces of a Hilbert space. In fact, this construction reproduces the Born rule for quantum probabilities. Quantum mechanical description can be made constructive if we replace the general group of unitary transformations of the Hilbert space with unitary representations of finite groups. It is known that any linear representation of a finite group is unitary and can be realized as a subrepresentation of some permutation representation. Thus, quantum mechanical problems can be formulated in terms of groups of permutations. This approach allows us to clarify the meaning of a number of physical concepts. In particular, the quantum randomness can be naturally explained by the fundamental impossibility to trace the individuality of indistinguishable objects in their evolution: only invariant relations among the objects are available in observations. The emergence of complex numbers in the formalism of quantum mechanics follows quite naturally from the general properties of finite groups. A finite quantum model is given by specifying a finite group and its unitary representation. An elementary step in the study of the model is the calculation of the probability distribution of transitions between adjacent observations. The probability of the entire trajectory is calculated as the product of the probabilities of the elementary transitions. An important task is the search for the most probable trajectories. This task is equivalent to the computationally simpler search for the trajectories with minimum entropy (negative logarithm of probability). The entropy of transition between adjacent observations and the entropy of the entire trajectory become, respectively, the Lagrangian and the action in the continuum approximation. Thus, the principle of selection of the most likely trajectories goes into the principle of least action in the continuum limit. In our study we use the Monte Carlo simulation and the system for computational group theory GAP.

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